MATH 337
INTRODUCTION TO SOLITON THEORY
Second Midterm Exam

April 27 , 2009
Monday 14.40-16.30, SAZ-19

PROBLEMS.

1. *Boussinesq equation.* a) Show that the bilinear form of the equation

$$u_{tt} - u_{xx} + 3(u^2)_{xx} - u_{xxxx} = 0$$

is

$$(D_t^2 - D_x^2 - D_x^4)(f \cdot f) = 0$$

where $u(x,t) = -2(\partial^2/\partial x^2) \log f$.
b) Using the above bilinear form find one soliton solution of the Boussinesq equation

2. Backlund transformations for the sine-Gordon equation $u_{xt} = \sin u$ are given as follows

$$(u+v)_x = 2\lambda \sin\left(\frac{u-v}{2}\right), \quad (u-v)_t = \frac{2}{\lambda} \sin\left(\frac{u+v}{2}\right)$$

where λ is an arbitrary constant (Backlund parameter)
a) Prove that the above relations define the auto-Backlund transformations for the sine-Gordon equation.
b) Use the above Backlund transformations to show that four solutions $u_i(x,t), (i = 0,1,2,3)$ are related by (nonlinear superposition)

$$\tan\left(\frac{u_3 - u_0}{4}\right) = \frac{\lambda_1 + \lambda_2}{\lambda_1 - \lambda_2} \tan\left(\frac{u_1 - u_2}{4}\right)$$

c) Find u_3 starting from the zero solution $u_0 = 0$.