MATH 337
INTRODUCTION TO SOLITON THEORY
Homework set 2

March 1, 2012
Due March 13 , 2012

PROBLEMS.

1. Find, if they exists, the eigenvalues and eigenfunctions of

$$\psi'' + (\lambda - U_0)\,\psi = 0$$

where U_0 is any real constant.

2. Find the eigenvalues and eigenfunctions of

$$\psi'' + (\lambda - u(x))\,\psi = 0$$

where

$$u(x) = \begin{cases} U_0, & 0 < x < 1 \\ \\ 0, & x < 0,\ x > 1 \end{cases}$$

where U_0 is a positive constant.

3. Find the eigenvalues and eigenfunctions of

$$\psi'' + (\lambda - u(x))\,\psi = 0$$

where $u(x) = -U_0\,\delta(x) - U_1\,\delta(x-1)$. Here U_0 and U_1 are positive constants, and show that there is only one one discrete eigenfunction if $(U_0 + U_1)/(U_0\,U_1) > 1$.

4. Find the discrete eigenfunctions for $N = 3$, where $u(x) = -N(N+1)\,sech^2 x$.