MATH 337
INTRODUCTION TO SOLITON THEORY
Homework set 1

February 13 , 2012
Due February 27 , 2012

PROBLEMS.

1. Modified Korteweg de Vries (mKdV) equation is given by

$$u_t + 6\rho\, u^2\, u_x + u_{xxx} = 0$$

where ρ is either 1 or -1. Show that the mKdV equation is invariant under the following group of transformations : In each case λ is a constant.
(i). $t \to t + \lambda,\ x \to x,\ u \to u$,
(ii).$t \to t,\ x \to x + \lambda,\ u \to u$,
(iii).$t \to \lambda^3 t,\ x \to \lambda x,\ u \to \lambda^{-1} u$, $(\lambda \neq 0.)$

2. Travelling wave solutions of the mKdV equation for $\rho = 1$ and for $\rho = -1$.
(i). Find the most general travelling wave solutions,
(ii). Find the solitary wave (rapidly decaying) solutions.

3. Use the transformation (iii) in the first question and find the group invariant solutions of the mKdV equation with respect to this group.