MATH 337
INTRODUCTION TO SOLITON THEORY
Second Midterm Exam

March 20 , 2012
Thursday 10.40-12.30, SAZ-20

QUESTIONS.

[35]**1**. Write a functional, $J[u] = \int_{-\infty}^{\infty} L dx$, so that Euler-Lagrange equation, $E(L) = 0$, is the modified KdV equation.

[35]**2**. Hirota bilinear form of the KdV equation, $u_t - 6uu_x + u_{xxx} = 0$, is $(D_t\, D_x + D^4)(f \cdot f) = 0$ where $u = -2\frac{\partial^2 \ln f}{\partial x^2}$. Here D_t and D_x are the Hirota differential operators. Find two soliton solutions of the KdV equation

[35]**3**. Let u satisfy the Sine-Gordon equation $u_{xt} = \sin u$. Show that the relations below among u and v are the auto Backlund transformations

$$(u+v)_x = 2\lambda \sin \frac{u-v}{2})$$
$$(u-v)_t = \frac{2}{\lambda} \sin \frac{u+v}{2})$$

for the sine-Gordon equation where v also satisfies the Backlund transformation and λ is the Backlund parameter. Find a solution for u when $v = 0$.