MATH 337
INTRODUCTION TO SOLITON THEORY
First Midterm Exam

March 20 , 2012
Tuesday 08.40-10.30, SAZ-20

PROBLEMS. Solve any three of the following four equations

[35]**1**. **(1.a)**. Solve the following initial value problem: $u_t + xu_x = 0$ with $u(x,0) = e^{-x}$. **(1.b)**. Consider the initial value problem

$$u_t + uu_x = 0, \quad x \in \mathbb{R}, \tag{1}$$

with

$$u(x,0) = \begin{cases} 1 - x^2, & |x| \leq 1 \\ 0, & |x| > 1 \end{cases}$$

Sketch the characteristic diagram. At what time t_b does the wave break? Find a formula for solution.

[35]**2**. Discuss the dispersion relation for the equation

$$\begin{aligned} &u_t + u_x - u_{xx} = 0, \\ &u_t + u_x + u_{xxx} = 0 \\ &u_t + u_x + u_{xxx} - u_{xx} = 0. \end{aligned} \tag{2}$$

Find the group and phase velocities of the above waves.

[35]**3**. Find a rapidly decaying (or solitary wave) solution of the equation $u_t + uu_x + u_{xxx} = 0$. Discuss the properties of the solution.

[35]**4**. Find the translational and scale symmetries of the equation $u_t + uu_x + uu_{xx} = 0$. Find the group invariant solutions.