MATH 343: METHODS OF APPLIED MATHEMATICS

Second Midterm Exam:

13 December 2000 Wednsday 18.00-22.00 SAZ:13

QUESTIONS: Solve at least four questions.

(25)**1.a**. Find the singular points of the linear ordinary differential equation

$$z^2\,(z^2-1)\,\frac{d^2u}{dz^2}+z\,(2z^2-1)\,\frac{du}{dz}+u=0 \tag{1}$$

and give the series expansion of $u(z)$ aboit its regular singular. Do not determine the coefficints C_n's.

1.b Expres the solution $u(z)$ of the above differential equation interms of the Riemann P-symbol and the hypergeometric function.

(25)**2**. Find an integral representation of the Legendre function $P_\lambda(z)$ satisfying

$$(1-z^2)\,\frac{d^2u}{dz^2}-2z\,\frac{du}{dz}+\lambda(\lambda+1)\,u=0 \tag{2}$$

where λ is a real number. Discuss the case where $\lambda=n$ a non-negative integer.

(25)**3**. Find all possible second order linear ordinar differential equations having two regular singular points.

(25)**4.a** Find the singular points of the differential equation

$$z(1-z)\,\frac{d^2u}{dz^2}-2\frac{du}{dz}+2u=0 \tag{3}$$

and classify them. Find the indices of each regular singular points

4.b Find the series solution of this equation about all its regular singular points.

4.c Find the most general solution of this equation.

(25)**5.a**. Express $\frac{d^m}{dz^m} F(\alpha, \beta; \beta; z)$ interms of elementary functions. Here m is a non-negative integer and F is the hypergeometric function.

5.b Determine also $F(1, 1; 2; z)$ interms of elementay functions.

5.c Find $\frac{d^n}{dz^2} F(a, b; c; z)$ interms F-function itself.