MATH 240- DIFFERENTIAL EQUATIONS
Homework Set I

due June 21, 2002 Friday 08:40

QUESTIONS:

1. Consider the initial value problem

$$y' - \frac{3}{2}y = 3t + 2e^t, \quad y(0) = y_0$$

Find the value of y_0 that separatessolutions that grow positively as $t \to \infty$ from those that grow negatively. How does the solution that corresponds to this critical value of y_0 behave as $t \to \infty$?.

2. Show that if a and λ are positive constants, and b is any real number, then every solution of the equation

$$y' + ay = be^{-\lambda t}$$

has the property that $y \to 0$ as $t \to \infty$. *Hint*: Consider the cases $a = \lambda$ and $a \neq \lambda$ sepaartely.

3. Solve the initial value problem

$$y' + p(t)y = 0, \quad y(0) = 1$$

where $p(t) = 2$, for $0 \leq t \leq 1$ and $p(t) = 1$ for $t > 1$.

4. Let

$$y' = (1 - 2x)/y, \quad y(1) = -2$$

(a) Find the solution in explicit form
(b) Plot the graph of the solution
(c) Determine (at least approximately) the interval in which the solution is defined

5. Let

$$y' = \frac{t - y}{2t + 5y}$$

State the region in the ty plane where the hypothesis of the Theorem 2.4.1 are satisfied. Thus there is a unique solution through each given initial point in thus region

6. Let

$$y' = ty(3 - y)$$

(a) Draw a direction field and plot (or sketch) several solutions of given differential equation.
(b) Describe how solutions appear to behave as t increases. and how their behavior depneds on the initial value y_0 when $t = 0$.