MATH 225: DIFFERENTIAL EQUATIONS AND LINEAR ALGEBRA

Second Midterm Exam
April 25, 2006
17:40 - 19:40

Name : _____

ID# : _____

Department : _____

Instructor : Metin Gürses

- **The exam consists of 4 questions of equal weight.**
- Please read the questions carefully.
- **Show all your work in legibly written, well-organized mathematical sentences.**
- Calculators and dictionaries are not allowed.
- **Simplify as far as possible unless otherwise stated.**
- Please turn off your cellular phones before the exam starts.

Please do not write below this line.

Q1	Q2	Q3	Q4	TOTAL
25	25	25	25	100

1. Prove the following statements: **1.a)** Let A be an $n \times n$ invertible matrix. Then the homogeneous linear system $A\vec{x} = 0$ has only the trivial solution. **1.b)** A set of vectors in a vector space containing the zero vector can not be linearly independent. **1.c)** Let $\{\vec{v}_1, \vec{v}_2, \cdots \vec{v}_k\}$ be a set of orthogonal vectors in $\mathbb{R}^n$ $(k \leq n)$. Then they are linearly independent $(k \leq n)$

2.a) A subset V of $\mathbb{R}^4$ is defined by means of the condition $w = x + y + z$ on a typical vector (x, y, z, w). Prove that V is a subspace of $\mathbb{R}^4$ and find a basis of V.

2.b) Let $P_3(x)$ be set of all polynomials of degree 3 on an interval I. Prove that $P_3(x)$ is a vector space of dimension 4.

3a.) Let V be a subspace of $\mathbb{R}^4$ spanned by the vectors $\vec{v}_1 = (2, 5, 5, 4, 3)$ and $\vec{v}_2 = (3, 7, 8, 8, 8)$. Find a basis for the orthogonal compliment V^+ of V.

3b.) Solve the initial value problem

$$y'' - 6y' + 25y = 0$$

with $y(0) = 0$ and $y'(0) = 4$.

4.a) Solve the following linear system by the use of Cramer's rule

$$\begin{aligned} 3x_1 + 4x_2 - 3x_3 &= 5, \\ 3x_1 - 2x_2 + 4x_3 &= 7, \\ 3x_1 + 2x_2 - x_3 &= 3, \end{aligned}$$

4.b). Let

$$A = \begin{bmatrix} x^2 - 1 & x - 1 & x^2 - 3x + 2 \\ \frac{3}{4}\,x^2 & \frac{1}{2}\,x & 0 \\ 12 & 2 & -12 \end{bmatrix}, \tag{1}$$

Find at least three values of x where the determinant of A vanishes.