MATH 225: DIFFERENTIAL EQUATIONS AND LINEAR ALGEBRA

First Midterm Exam
March 13, 2006
17:40 - 19:40

Name : _____

ID# : _____

Department : _____

Instructor : Metin Gürses

- **The exam consists of 4 questions of equal weight.**
- Please read the questions carefully.
- **Show all your work in legibly written, well-organized mathematical sentences.**
- Calculators and dictionaries are not allowed.
- **Simplify as far as possible unless otherwise stated.**
- Please turn off your cellular phones before the exam starts.

Please do not write below this line.

Q1	Q2	Q3	Q4	TOTAL
25	25	25	25	100

1.a) Show that the function $y(x) = A + Be^x + Ce^{2x}$ satisfies the differential equation $y''' - 3y'' + 2y' = 0$, where A, B, and C are arbitrary constants.

1.b) Find the constants A, B, and C so that $y(0) = 1$, $y'(0) = -1$, $y''(0) = 0$.

2.a) Solve the following system by Gauss-Jordan Elimination

$$\begin{aligned} x_1 - 2x_2 + x_3 - 4x_4 &= 1, \\ x_1 + 3x_2 + 7x_3 + 2x_4 &= 2, \\ x_1 - 12x_2 - 11x_3 - 16x_4 &= -1 \end{aligned}$$

2.b) What conditions must b_1,b_2, and b_3 satisfy in order for the system

$$\begin{aligned} x_1 + x_2 + 2x_3 &= b_1, \\ x_1 \quad\quad + x_3 &= b_2, \\ 2x_1 + x_2 + 3x_3 &= b_3 \end{aligned}$$

to be consistent. Find the solutions.

3.a) Solve the differential equation $x^2y' + 2xy = 5y^4$ with $y(1) = 1$. Discus the existence and uniqueness of the solution of the differential equation with the initial condition $y(0) = 0$.

3.b) Solve the differential equation $(x + \ln y)dx + (\frac{x}{y} + e^y)dy = 0$ with $y(0) = 1$

4.a) Find the Reduced Row-Echelon matrix of the augmented coefficient matrix of the following system. Then find the solution of the system.

$$\begin{aligned} x_1 + 2x_2 + 3x_3 &= 4, \\ 2x_1 + 5x_2 + 3x_3 &= 5, \\ x_1 \quad\quad + 8x_3 &= 9 \end{aligned}$$

4.b) Let $XA = Z$ where A, Z are matrices given by

$$A = \begin{bmatrix} 0 & -2 & 3 \\ 1 & 2 & -3 \\ 2 & 0 & 4 \end{bmatrix}, \quad Z = \begin{bmatrix} 4 & -2 & 3 \\ -1 & 2 & 0 \end{bmatrix}.$$

Find A^{-1} and the matrix X.