MATH 543: METHODS OF APPLIED MATHEMATICS

First Midterm Exam:

30 October 2000 Monday 18.30-20.30 SAZ:13

QUESTIONS: Solve at least four questions.

(25)**1.a**. Let $C_{(n)}(x)$ be a classical polynomial defined in $[a,b]$ with a weight function $w(x)$. Let

$$C_{(n)}(x) = k_n\, x^n + k'_n\, x^{n-1} + \cdots,$$

and

$$h_n = \int_a^b w(x)\, [C_{(n)}(x)]^2\, dx.$$

a). Find $\int_a^b w(x)\, C_{(n)}(x)\, C_{(m)}(x)\, dx$.

c). Let $C_{(n)}(x) = H_n(x)$, *the Hermite Polynomial.* Prove that

$$\frac{d^m}{dx^m}\, H_n(x) = 2^m\, \frac{n!}{(n-m)!}\, H_{n-m}(x)$$

(25)**2**. **a)**. Prove that

$$\delta(f(x)) = \sum_1^m \frac{1}{|f'(x_k)|}\, \delta(x - x_k)$$

where x_k's are the roots of f in the interval on which f is defined.

b). Find

$$\int_{-\infty}^{\infty} \delta(x^2 - \pi^2)\, \cos x\, dx,$$

$$\int_{0.5}^{\infty} \delta(sin(\frac{\pi x}{2})(1/3)^x\, dx$$

c). Show that the sequence of functions $\{\frac{\sin nx}{\pi x}\}$ approaches $\delta(x)$.

(25)**3**. Solve $u'' + u' = f(x)$ with the boundary conditions

a). $u(0) = 0$ and $u(\infty) = 0$

b). $u(0) = u\infty$ and $u'(\infty) = 0$.

(25)**4.a**. Let $u'' + \alpha(x)\, u' + \beta(x)u = 0$ in $[a, b]$ with the boundary conditions $u(a) = u(b) = 0$ and $\beta = \frac{1}{2}\alpha'$. Prove that $u(x) = 0$ for all $x \in [a, b]$.

b) Find the Fourier transform of $\frac{1}{(1+x^2)^2}$.

c) Find the Fourier series of the periodic function $f(x)$ with the period $2a$, given in a period a $f(x) = x,\ x \in (-a, a)$