MATH116: QUIZ VIII

Name:
Student Number:
Section: 2
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Question:
Use the transformation $x = u + \frac{1}{2}\,v, \quad y = v$ to evaluate the integral

$$\int_0^2 \int_{y/2}^{(y+4)/2} y^3\,(2x-y)\;e^{(2x-y)^2}\,dx\,dy$$

Solution:
Transformation is

$$x = u + \frac{1}{2}\,v, \quad y = v$$

or

$$u = x - y/2, \quad v = y$$

Then the Jacobian $J = 1$ and the region G in the $uv-$ coordinates is the rectangle $0 \le u \le 2, \quad 0 \le v \le 2$ Then

$$\int_0^2 \int_{y/2}^{(y+4)/2} y^3\,(2x-y)\;e^{(2x-y)^2}\,dx\,dy = \int_0^2 \int_0^2 v^3\,ue^{u^2}\,du\,dv$$

$$= (\int_0^2 v^3 dv)(\int_0^2 ue^{u^2} du) = 2(e^4 - 1)$$