MATH116: QUIZ IV

Name:
Student Number:
Section: 2
Date: June 15, 2006

Question
Find all local maximum, minimum and saddle points of the function

$$f(x,y) = x^3 - 6xy + 3y^2$$

Solution

$$f_x = 3x^2 - 6y = 0, \quad f_y = 6y - 6x = 0$$

From the second equation we obtain $y = x$. Using this information in the first one we get $x = 0$ and $x = 2$. Hence we have two points $(0,0)$ and $(2,2)$.

For the point $(0,0)$ we have $f_{xx}(0,0) = 0, f_{yy}(0,0) = 6, f_{xy}(0,0) = -6$. Hence $f_{xx}(0,0) \cdot f_{yy}(0,0) - (f_{xy}(0,0))^2 = -36 < 0$. Hence $(0,0)$ is a saddle point.

For the point $(2,2)$ we have $f_{xx}(2,2) = 12, f_{yy}(2,2) = 6, f_{xy}(2,2) = -6$. Hence $f_{xx}(2,2) > 0$ and $f_{xx}(2,2) \cdot f_{yy}(2,2) - (f_{xy}(2,2))^2 = 36 > 0$. Hence $(2,2)$ is a point where f has a local minimum value.