MATH116-1: QUIZ VII

Name:
Student Number:
Section: 1
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Question: Find the surface area of the cone $z = \sqrt{x^2 + y^2}$ with base of radius 5 cm.

a. Cone as a level surface of $f(x, y, z) = z - \sqrt{x^2 + y^2}$.

$$Area(cone) = \int \int_S d\sigma$$

where

$$d\sigma = \frac{dA}{\cos \gamma}$$

with $\cos \gamma = \mathbf{n} \cdot \mathbf{k}$ where

$$\mathbf{n} = \frac{\nabla f}{||\nabla f||} = \frac{1}{\sqrt{2}} (-2x\mathbf{i} - 2y\mathbf{j} + \mathbf{k})$$

Hence $\cos \gamma = \frac{1}{\sqrt{2}}$ Then

$$Area(cone) = \sqrt{2} \int \int_R dA = \sqrt{2} AreaR$$

where R is the disk with radius 5. Hence Area(cone)$= 25\sqrt{2}\ \pi$.

b. By using the parametric form of the cone: $\mathbf{r}(r, \theta) = r \cos \theta \mathbf{i} + r \sin \theta \mathbf{j} + r\mathbf{k}$. Then

$$\mathbf{r}_{,r} = \cos \theta \mathbf{i} + \sin \theta \mathbf{j} + \mathbf{k}$$

$$\mathbf{r}_{,\theta} = -r \sin \theta \mathbf{i} + r \cos \theta \mathbf{j}$$

Then $||\mathbf{r}_{,r} \times \mathbf{r}_{,\theta}|| = \sqrt{2}\, r$ Hence

$$Area(cone) = \int_0^{2\pi} \int_0^5 ||\mathbf{r}_{,r} \times \mathbf{r}_{,\theta}||\, dr d\theta = 25\sqrt{2}\ \pi$$