MATH116-1: QUIZ VI

Name:
Student Number:
Section: 1
Date: July 15, 2010

Question: Find the (counterclockwise) circulation of the vector field $\mathbf{F} = (x^3 - y)\mathbf{i} + (x - y^3)\,\mathbf{j}$ around the unit circle.

Solution
(a). Using GT. $M = x^3 - y,\ N = x - y^3$. Let R br the disk bounded by the unit circle. Then

$$circ(F) = \oint_C \mathbf{F}\cdot d\mathbf{r} = \oint_C (M\dot{x} + N\dot{y})\,dt = \int\int_R (N_{,x} - M_{,y})dA = 2\,A(R) = 2\pi$$

(b). Calculation of the line integral directly. C is parametrized by $x = \cos t,\ y = \sin t$ where $0 \le t \le 2\pi$. Then

$$circ(F) = \oint_C \mathbf{F}\cdot d\mathbf{r} = \oint_C (M\dot{x} + N\dot{y})\,dt = \int_0^{2\pi} [-\sin t\,(\cos^3 t - \sin t) + \cos t\,(\cos t - \sin^3 t)]dt$$

$$= \int_0^{2\pi} [1 - \sin t \cos t]dt = 2\pi$$