MATH116: QUIZ V

Name:
Student Number:
Section: 1
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Question: Evaluate the triple integral

$$\int\int\int_D \sqrt{x^2+y^2}\, dV.$$

Here the region D is the ball $x^2+y^2+z^2 \leq a^2$ where a is a positive number.

Solution: (a) **Using the spherical coordinates:** (easier)
$x = \rho\cos\theta\sin\phi,\ y = \rho\sin\theta\sin\phi,\ z = \rho\cos\phi$. Hence $\sqrt{x^2+y^2} = \rho\sin\phi$ then

$$\int\int\int_D \sqrt{x^2+y^2}\, dV = \int_0^{2\pi}\int_0^{\pi}\int_0^{a} (\rho\sin\phi)\,\rho^2\,\sin\phi\, d\rho\, d\phi\, d\theta$$

$$= \int_0^{2\pi}\int_0^{\pi}\int_0^{a} \rho^3\sin^2\phi)\, d\rho\, d\phi\, d\theta = \frac{2\pi\, a^4}{4}\int_0^{\pi}\sin^2\phi d\phi = \frac{\pi^2\, a^4}{4}$$

(b) **Using the Cartesian coordinates directly:**(More difficult than (a))
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$$\int\int\int_D \sqrt{x^2+y^2}\, dV = \int\int_R \int_{-\sqrt{a^2-x^2-y^2}}^{\sqrt{a^2-x^2-y^2}} \sqrt{x^2+y^2}\, dz dA$$

where R is the disk in the xy-plane with radius a. Then

$$\int\int\int_D \sqrt{x^2+y^2}\, dV = 2\int\int_R \sqrt{x^2+y^2}\,\sqrt{a^2-x^2-y^2}\, dA$$

Using the polar coordinates we get

$$\int\int\int_D \sqrt{x^2+y^2}\, dV = 2\int_0^{2\pi}\int_0^{a} r^2\,\sqrt{a^2-r^2} dr d\theta = \pi\int_0^{\pi/2}\sin^2 2\alpha\, d\alpha = \frac{\pi^2\, a^4}{4}$$

In the last integral we made use of the change of variables $r = a\sin\alpha$.