MATH116: QUIZ-III

Name:
Student Number:
Section:
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Question
Find the maximum and minimum values of the function $f(x, y, z) = x^2 - 2y^2 + 2y + z + 3$ on the piece of the curve $x = 2t - 1$, $y = -t + 1$, $z = t^2$ where $-1 \leq t \leq 1$.
Solution: $f(x, y, z)$ restricted on the curve

$$w(t) = f(x(t), y(t), z(t)) = (2t-1)^2 - 2(-t+1)^2 + 2(-t+1) + t^2 + 3 = 3t^2 - 2t + 4$$

where $t \in [-1, 1]$.
Interior points:

$$w' = 6t - 2 = 0,$$

hence $t = 1/3$, and $w(1/3) = 11/3$. This means

$$f(-1/3, 2/3, 1/9) = 11/3$$

End points: $w(-1) = 9$ or

$$f(-3, 2, 1) = 9$$

and $w(1) = 5$ or

$$f(1, 0, 1) = 5$$

Then the maximum value is $f(-3, 2, 1) = 9$ and the minimum value is $f(-1/3, 2/3, 1/9) = 11/3$ on the given curve.