MATH116: QUIZ-II

Name:
Student Number:
Section:
Date: June 18, 2010

Question
Suppose that $f(x,y,z)=\frac{1}{2}(x^2+y^2)+\frac{1}{2}z^3+\frac{1}{2}z^2-2z$. (a) Find the directional derivative of f at $(-1,1,0)$ in the direction $3\,\mathbf{i}+2\,\mathbf{j}-6\mathbf{k}$
(b) Find the equations of tangent plane and the normal line of the level surface $f(x,y,z)=1$ at the point $P(1,1,0)$.
Solution: (a) f is differentiable everywhere, hence

$$(D_u f)_P = \mathbf{u}\cdot(\nabla f)(P)$$

where

$$\mathbf{u}=\frac{1}{7}(3\,\mathbf{i}+2\,\mathbf{j}-6\mathbf{k}),$$
$$(\nabla f)(P)=-\mathbf{i}+\mathbf{j}-2\mathbf{k}$$

Then

$$(D_u f)_P=\frac{1}{7}(3\,\mathbf{i}+2\,\mathbf{j}-6\mathbf{k})\cdot(-\mathbf{i}+\mathbf{j}-2\mathbf{k})=\frac{11}{7}$$

(b)
Tangent Plane: $-(x-1)+(y-1)-2z=0$ After simplification

$$-x+y-2z=0$$

Normal Line: $x=1+t,\ y=1-t,\ z=-2t$ where $t\in\mathbb{R}$.