MATH116-01 QUIZ 1

Name:
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Question. Prove the following limit.

$$\lim_{(x,y)\to(0,0)} \left(\frac{x^2\, y^2 \sin x}{x^2 + y^6} \right) = 0$$

Solution:

$$|f(x,y) - L| = \left| \frac{x^2\, y^2 \sin x}{x^2 + y^6} \right| \leq \left| \frac{x^2\, y^2}{x^2 + y^6} \right| \leq \left| \frac{x^2}{x^2 + y^6} \right| y^2 \leq y^2 \leq x^2 + y^2 < \delta^2 = \epsilon$$

Hence there exists a δ for a given ϵ by $\delta = \sqrt{\epsilon}$. For all x and y in $0 < \sqrt{x^2 + y^2} < \delta$ implies $|f(x,y) - L| < \epsilon$.