HOMEWORK III (MATH 102)(section 2): *March 24, 2004*

1. Page 681 exercises 37 and 38

37. How close is the approximation $\sin x = x$ when $|x| \leq 10^{-3}$? For which of these values of x is $x < \sin x$?.

38., The estimate $\sqrt{1+x} = 1 + (x/2)$ is used when x is small. Estimate the error when $|x| \leq 0.01$.

2. Page 691 exercises 38 and 40

38). Use series to evaluate the limit

$$lim_{t\to 0}\frac{1-\cos t-(t^2/2)}{t^4}$$

noindent **40)**. Use series to evaluate the limit

$$lim_{y\to 0}\frac{\tan^{-1} y-\sin y}{y^3\cos y}$$

3. Page 691 exercise 49

49). Derive the series

$$\tan^{-1} x = \frac{\pi}{2} - \frac{1}{x} + \frac{1}{3x^3} - \frac{1}{5x^5} + \cdots \quad x > 1$$

$$\tan^{-1} x = -\frac{\pi}{2} - \frac{1}{x} + \frac{1}{3x^3} - \frac{1}{5x^5} + \cdots \quad x < -1$$

4. Page 708 exercises 36 and 38

36). Does the series converge, converge conditionally or diverge? Give reasons for your answers

$$\sum_{n=1}^{\infty} \frac{(-1)^n\,(n^2+1)}{2n^2+n-1}$$

38. The same question as the above one

$$\sum_{n=1}^{\infty} \frac{2^n\,3^n}{n^n}$$

5. Page 709 exercise 61 and 64

61). Find the Maclaurin series of $\cos(x^{5/2})$

64). Find the Maclaurin series of e^{-x^2}

6. Page 710 exercise 105, 106

105). If $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ are convergent series of nonnegative numbers, can anything be said about $\sum_{n=1}^{\infty} a_n b_n$? Give reasons for your answer.

106). If $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ are divergent series of nonnegative numbers, can anything be said about $\sum_{n=1}^{\infty} a_n b_n$? Give reasons for your answer.

7. Page 711 exercise 107, 108

107. Prove that the sequence $\{x_n\}$ and the series $\sum_{k=1}^{\infty} (x_{k+1} - x_k)$ both converge or both diverge.

108. Prove that $\sum_{n=1}^{\infty} (a_n/(1+a_n))$ converges if $a_n > 0$ for all n and $\sum_{n=1}^{\infty} a_n$ converges.