HOMEWORK II (MATH 102)(section 2): *March 10, 2004*

1. Page 617 exercises 54 and 56

Which of the sequences $\{a_n\}$ converge, which diverge? Find the limit of each convergent sequence **54)**. $a_n = (n^2+n)^{1/n}$, **56)**. $a_n = n - \sqrt{n^2-n}$

2. Page 618 exercises 71 and 72

71). Prove that if $\{a_n\}$ is a convergent sequence, then to every positive integer ϵ there corresponds an integer N such that for all m and n

$$m > N \text{ and } n > N \Longrightarrow |a_n - a_m| < \epsilon$$

72). Prove that limits of sequences are unique. That is, show that if L_1 and L_2 are the numbers such that $a_n \to L_1$ and $a_n \to L_2$, then $L_1 = L_2$.

3. Page 626 exercise 9.a

9.a). The following squence come from the recursion formula for Newton's method (see exercise 7). Does the sequence converge? If it so, to what value? Begin by identifying the function that generates the sequence.

$$x_0 = 1, \quad x_{n+1} = x_n - \frac{x_n^2 - 2}{2x_n} = \frac{x_n}{2} + \frac{1}{x_n}$$

4. Page 638 exercises 56 and 58

56). For what values of r does the the infinite series

$$1 + 2r + r^2 + 2r^3 + r^4 + 2r^5 + r^6 + \cdots$$

converge? Find the sum of the series when it converges.

58). Find geometric series $A = \sum a_n$ and $B = \sum b_n$ that illustrate that $\sum a_n\, b_n$ may converge without being equal to AB.

5. Page 650 exercise 68 and 70

68). If $\sum a_n$ is convergent series with nonnegative numbers, can anything be said about $\sum(a_n/n)$?

70). Prove that if a_n is a convergent series of nonnegative terms, then $\sum(a_n)^2$ converges.

6. Page 650 exercise 76.d

76.d). By studying exercise 75 determine whether the infinite series

$$\sum \frac{1}{n(\ln n)^3}$$

converges or diverges.

(Exercise 75 is about the convergence of the series

$$\sum \frac{1}{n(\ln n)^p}$$

where p is a positive number)