Math 101
First Midterm Exam
March 12, 2011
12.30 - 14:30

Name : _____

ID# : _____

Department : _____

Section : _____

- The exam consists of 5 questions of equal weight.
- Read the questions carefully.
- **Solutions, not answers, get points.** Show all your work in well-organized mathematical sentences. Explain your reasoning carefully and in full.
- **What can not be read will not be read.** Write clearly and cleanly.
- Simplify your answers as far as possible unless otherwise stated.
- Calculators and dictionaries are not allowed.
- Turn off all electronic devices including your cell phones before the exam starts.

Please do not write below this line.

Q1	Q2	Q3	Q4	Q5	TOTAL
20	20	20	20	20	100

1a. Show that

$$\lim_{x\to 0^+} \sqrt{x}\, e^{\cos\frac{\pi}{x}} = 0$$

1b. Find the limit

$$\lim_{x\to +\infty} \left[x\sqrt{3x^2+4e^{-x}+1} - x\,\sqrt{3x^2+2\,e^{-x}}\ \right]$$

2a. The following function is differentiable everywhere

$$f(x) = \begin{cases} \sin(ax) + b, & x < 0 \\ \sin^2(2x) + 2x, & x \geq 0. \end{cases}$$

Find the constants a and b.

2b. Find the derivative of the function $y = 2(\ln x)^{x/2}$.

3a. Find the tangent and the normal lines to the curve $x^2 + y^2 = y^4 + 1$ at the point (1,1).

3b. Suppose that the differentiable function $y = f(x)$ has an inverse and that the graph of $f(x)$ passes through the point (1,4) with slope 2. Find the slope of the the graph of $f^{-1}(x)$ at (4,1).

4a. Find the absolute maximum and absolute minimum values of the function

$$f(x) = \frac{x+1}{x^2+2x+2}$$

on the segment [-4,1].

4b. Show that the function $f(x) = x^3 + \frac{4}{x^2} + 7$ has exactly one zero in the interval $(-\infty, 0)$.

5a. Sand falls from a conveyor belt at the rate of 10 m^3/min onto the top of a conical pile. The height of the pile is always three-eights of the base diameter. How fast are the **(a)** height and **(b)** radius changing when the pile is 4 m high?

5b. Show that the function $y = 2\sin(\ln x)$ satisfies the equation $x^2y'' + xy' + y = 0$.