

MATH 101, ALL SECTIONS, HOMEWORK #5 (SPRING 2011)

Due to the week starting May 9, at the first hour of the last lecture day that week.

QUESTION 1.

- (a) Find the length of the curve

$$y = \ln \cos x, \quad 0 \leq x \leq \frac{\pi}{4}.$$

- (b) Find the area of the surface generated by rotation of the curve

$$y = \frac{e^x + e^{-x}}{2}, \quad 0 \leq x \leq \ln 2$$

about x -axis.

QUESTION 2. Evaluate the following integrals by an appropriate substitution and/or integration by parts.

(a) $\int x^3 \ln x dx;$

(b) $\int \sin(\ln x) dx;$

(c) $\int x(\ln x)^2 dx;$

QUESTION 3. Evaluate the following integrals

(a) $\int \cos 3x \sin 2x \cos x dx;$

(b) $\int \sin^2 x \cos^2 x dx;$

(c) $\int x\sqrt{1 - \cos x} dx;$

(d) Prove the reduction formula $\int \tan^n x dx = \frac{1}{n-1} \tan^{n-1} x - \int \tan^{n-2} x dx;$

QUESTION 4. Evaluate the integrals

(a) $\int \frac{x^2 dx}{\sqrt{1+x^2}};$

(b) $\int \frac{dx}{x\sqrt{x^2-1}};$

(c) $\int_0^{\pi/6} \sqrt{1 + \sin x} dx$

QUESTION 5. Evaluate the integrals

(a) $\int \frac{dx}{x^3 + x^2 - 2x}$;

(b) $\int \frac{x^4 + 1}{x(x^2 + 1)^2} dx$.

and investigate convergence of the improper integrals

(c) $\int_0^{\infty} \sin(x^2) dx$;

Hint: Change the variable and integrate by parts.

(d) $\int_1^{\infty} \frac{x^3 dx}{x^4 + 2x^2 + 6}$.