

MATH101: HOMEWORK IV: Spring 2011

For all Sections

(Due April 18th week: first hour of the second lecture day)

QUESTIONS:

1. Evaluate the following indefinite integrals:

(1.a)

$$\int \frac{1}{x^3} \sqrt{\frac{1-x^2}{x^2}} dx$$

(1.b)

$$\int \frac{dy}{\sqrt{e^{2y} - 1}}$$

2. Evaluate the following definite integrals:

(2.a)

$$\int_0^{\pi/4} \pi^{\tan x} \sec^2 x dx$$

(2.b) Show that

$$2 \leq \int_{-1}^1 \sqrt{1+x^2} dx \leq 2\sqrt{2}$$

3. (3.a) Find the area of the region in the first quadrant bounded on the left by the y -axis, below by the curve $x = 2\sqrt{y}$, above left by the curve $x = (y-1)^2$, and above right by the line $x = 3 - y$ (*This is from the text book, page 352, problem 106*). (3.b) Graph the function

$$f(x) = \begin{cases} \sqrt{1-x}, & 0 \leq x < 1 \\ (7x-6)^{-1/3}, & 1 \leq x \leq 2 \end{cases}$$

and integrate it over its domain.

4. (4.a) Find the limit $\lim_{n \rightarrow \infty} \left(\frac{n}{n^2+1} + \frac{n}{n^2+4} + \cdots + \frac{n}{n^2+k^2} + \cdots + \frac{n}{2n^2} \right)$. (4.b) Find the volume of the following solid: The base of the solid is the disk $x^2 + y^2 \leq 1$. The cross-sections by the planes perpendicular to the y -axis between $y = -1$ and $y = 1$ are isosceles right triangles with one leg in the disk.

5. (5.a) Find the area between the curves $y^2 - x - 4y = 0$ and $y^2 + x - 2y = 0$. (5.b) Find the volume of the solid obtained by rotating the region bounded by $y = x - x^2$ and $y = 0$ about the line $x = 2$.