

MATH 101: HOMEWORK 3: Spring 2011

For all Sections

(Due on the week of March 28: first hour of the last lecture day)

1a. Let $f(x) = x^{2/3}(x^2 - 4)$. Find the open intervals on which f is increasing and decreasing and identify the extrema of f and the points where they occur.

1b. Let $f(x) = x^{2/3}(x - 5)$. Find the domain, possible symmetries, intervals of increasing and decreasing, critical points, extrema, intervals of concavity, points of inflection, and asymptotes. Sketch the graph.

2a. Find the limit $\lim_{x \rightarrow +\infty} \left(\frac{x^2 + 1}{x + 2} \right)^{1/x}$.

2b. Find the values of parameters a and b such that

$$\lim_{x \rightarrow 0} \left(\frac{\tan 2x}{x^3} + \frac{a}{x^2} + \frac{\sin bx}{x} \right) = 0.$$

3. The stiffness S of a rectangular beam is proportional to its width times the cube of its height. Find the dimensions of the stiffest beam that can be cut from a cylindrical log of diameter d .

4a. Use known formulas for areas to evaluate $\int_{-4}^0 \sqrt{16 - x^2} dx$.

4b. Find the values $a < b$ that minimize the integral $I(a, b) := \int_a^b (x^4 - 2x^2) dx$.

5a. Compute $\int_{-1}^1 (x^2 - 2x + 3) dx$.

5b. Compute $\int_0^\pi \frac{1}{2}(\cos x + |\cos x|) dx$.