

**MATH 101: HOMEWORK 3: Spring 2011**

**For all Sections**

(Due on the week of March 28: first hour of the last lecture day)

**1a.** Let  $f(x) = x^{2/3}(x^2 - 4)$ . Find the open intervals on which  $f$  is increasing and decreasing and identify the extrema of  $f$  and the points where they occur.

**1b.** Let  $f(x) = x^{2/3}(x - 5)$ . Find the domain, possible symmetries, intervals of increasing and decreasing, critical points, extrema, intervals of concavity, points of inflection, and asymptotes. Sketch the graph.

**2a.** Find the limit  $\lim_{x \rightarrow +\infty} \left( \frac{x^2 + 1}{x + 2} \right)^{1/x}$ .

**2b.** Find the values of parameters  $a$  and  $b$  such that

$$\lim_{x \rightarrow 0} \left( \frac{\tan 2x}{x^3} + \frac{a}{x^2} + \frac{\sin bx}{x} \right) = 0.$$

**3.** The stiffness  $S$  of a rectangular beam is proportional to its width times the cube of its height. Find the dimensions of the stiffest beam that can be cut from a cylindrical log of diameter  $d$ .

**4a.** Use known formulas for areas to evaluate  $\int_{-4}^0 \sqrt{16 - x^2} dx$ .

**4b.** Find the values  $a < b$  that minimize the integral  $I(a, b) := \int_a^b (x^4 - 2x^2) dx$ .

**5a.** Compute  $\int_{-1}^1 (x^2 - 2x + 3) dx$ .

**5b.** Compute  $\int_0^\pi \frac{1}{2} (\cos x + |\cos x|) dx$ .