

MATH 101, ALL SECTIONS, HOMEWORK #2 (SPRING 2011)

Due to the week starting February 28, at the first hour of the last lecture day that week.

QUESTION 1.

- (a) Find an equation of the tangent to the curve $y = \sqrt{x}$ at the point $(x_0, y_0) = (1, 1)$.
- (b) The height above the ground of a stone dropped by Galileo at $t = 0$ from the top of the Leaning Tower of Pisa varies with the time t measured in seconds by the equation $h = 56 - 4.9t^2$ meters. What is the speed of the stone (i) after two seconds; (ii) at the moment the stone hits the ground.

QUESTION 2. Apply the differentiation rules to find the derivatives of the following functions

- (a) $f(s) = \frac{\sqrt{s} - 1}{\sqrt{s} + 1}$;
- (b) $r = \frac{e^t}{t}$;
- (c) $u = \sin^{-1}(\ln x)$;
- (d) $v = \ln(\tan x)$.

QUESTION 3. Make use of the logarithmic derivative to differentiate the following functions:

- (a) $y = t^{\sqrt{t}}$;
- (b) $y = (\sin x)^x$;

QUESTION 4. Find derivatives $\frac{dy}{dx}$ of the functions given by implicit equations

- (a) $\ln xy = e^{x+y}$;
- (b) $x^y = y^x$.

QUESTION 5. Show that

- (a) $\lim_{n \rightarrow \infty} \left(1 + \frac{x}{n}\right)^n = e^x$;
- (b) The function $y = \sin(\ln x)$ satisfies the equation $x^2 y'' + xy' + y = 0$.