MATH101: HOMEWORK I: Spring 2011
For all Sections
(Due February 14th week: first hour of the last lecture day)

QUESTIONS:
1. (a) prove that

$$\lim_{x\to 0} \frac{|x|}{x}$$

does not exists.
(b). Show that

$$\lim_{x\to 0} \sqrt{x^3+x^2}\,\sin(\frac{\pi}{x}) = 0$$

2. (a). If $1 \le f(x) \le x^2+2x+2$ for all x find

$$\lim_{x\to -1} f(x)$$

(b). Show that

$$\lim_{x\to 0} \sqrt{x}\, e^{\cos\frac{\pi^2}{x}} = 0$$

3. (a). Find

$$\lim_{x\to 1} \arcsin \frac{1-\sqrt{x}}{1-x}$$

(b). Let

$$f(x) = \begin{cases} kx+1, & x \le 3 \\ kx^2+k, & x > 3 \end{cases}$$

be a continuous function for all x values, where k is a real constant. Find k.
4. Find the following limits
(a)

$$\lim_{x\to -\infty} \tan^{-1}(x^4-3x^3)$$

(b)

$$\lim_{x\to \pm\infty} \ln(\sqrt{2x^2+4x}-\sqrt{2x^2+2x})$$

5. Suppose that a function f is continuous on the closed interval $[0,1]$ and that $0 \le f(x) \le 1$ for every x in $[0,1]$. Show that there exists a number c in $[0,1]$ such that $f(c) = c$ (c is called a fixed point of f).