

MATH101: HOMEWORK I: Spring 2011

For all Sections

(Due February 14th week: first hour of the last lecture day)

QUESTIONS:

1. (a) prove that

$$\lim_{x \rightarrow 0} \frac{|x|}{x}$$

does not exist.

(b). Show that

$$\lim_{x \rightarrow 0} \sqrt{x^3 + x^2} \sin\left(\frac{\pi}{x}\right) = 0$$

2. (a). If $1 \leq f(x) \leq x^2 + 2x + 2$ for all x find

$$\lim_{x \rightarrow -1} f(x)$$

(b). Show that

$$\lim_{x \rightarrow 0} \sqrt{x} e^{\cos \frac{\pi^2}{x}} = 0$$

3. (a). Find

$$\lim_{x \rightarrow 1} \arcsin \frac{1 - \sqrt{x}}{1 - x}$$

(b). Let

$$f(x) = \begin{cases} kx + 1, & x \leq 3 \\ kx^2 + k, & x > 3 \end{cases}$$

be a continuous function for all x values, where k is a real constant. Find k .

4. Find the following limits

(a)

$$\lim_{x \rightarrow -\infty} \tan^{-1}(x^4 - 3x^3)$$

(b)

$$\lim_{x \rightarrow \pm\infty} \ln(\sqrt{2x^2 + 4x} - \sqrt{2x^2 + 2x})$$

5. Suppose that a function f is continuous on the closed interval $[0, 1]$ and that $0 \leq f(x) \leq 1$ for every x in $[0, 1]$. Show that there exists a number c in $[0, 1]$ such that $f(c) = c$ (c is called a fixed point of f).