Math 101
Final Exam
May 23, 2011
15:30 - 17:30

Name : _____

ID# : _____

Department : _____

Section : _____

Instructor : _____

- The exam consists of 5 questions of equal weight.
- Please read the questions carefully.
- **Show all your work in legibly written, well-organized mathematical sentences.**
- Calculators and dictionaries are not allowed.
- Simplify as far as possible unless otherwise stated.
- Please turn off your cellular phones before the exam starts.
- Extra sheets of paper, if needed, will be provided.

Please do not write below this line.

Q1	Q2	Q3	Q4	Q5	Total
20	20	20	20	20	100

1 a) Find the limit

$$\lim_{x \to 1} \frac{\sin^2(x^3 - 1)}{(x-1)^2}$$

1 b) Let

$$f(x) = \begin{cases} x^2, & x \geq 0 \\ -x^2, & x \leq 0. \end{cases}$$

Find $f'(x)$ and $f''(x)$ everywhere.

2 a) The line that is normal to the curve $x^2 + 2xy - 3y^2 = 0$ at $P(1, 1)$ intersects the curve at what other points?.

2 b) A rectangle has its base on the x-axis and its upper two vertices on the parabola $y = 12x - x^2$. What is the largest area that the rectangle can have and what are its dimensions?.

3 a) Find the area of the surface generated by revolving about the x-axis the curve $y = 2\sqrt{x-1}$ with $1 \leq x \leq 2$

3 b) Find the length of the curve $y = \sqrt{1-x^2}$ with $\frac{1}{\sqrt{2}} \leq x \leq 1$.

4 a) Find a formula for the nth partial sum of the series

$$\sum_{n=2}^{\infty} \frac{1}{n^2 - 1}$$

and use it to determine if the series converges or diverges. If the series converges, find its sum.

4 b) Evaluate

$$\int_0^{\infty} \frac{e^{-\sqrt{x}}}{\sqrt{x}}\, dx$$

5 a) Investigate the convergence of the series. Give reasons for your answer.

$$\sum_{n=1}^{\infty} \frac{e^{-\tan^{-1} n}}{1+n^2}$$

5 b) For which $a \geq 0$ does the series

$$\sum_{n=1}^{\infty} \frac{a^{(n^2)}}{(n)!}$$

converge? Give reasons for your answer.