**Question 1 [Work]:** A constant force, $F$, is applied to a block of mass $m$ on an inclined plane as shown in Figure 1. The block is moved with a constant velocity by a distance $s$. The coefficient of kinetic friction between the inclined plane and the block is $\mu_k$.

(a) Draw free-body diagrams and find the force $F$ (5 points).

(b) What is the net work done by $F$ (10 points)?

(c) Check case (a) if $\alpha=0$ and comment on the result (5 points).

![Figure 1](image)

**Solution 1:**

(a) From Newton’s 2nd law:

\[
\sum F_x = F \cos \alpha - f_k - mg \sin \theta = ma_x \quad (1)
\]
\[
\sum F_y = N - F \sin \alpha - mg \cos \theta = ma_y \quad (2)
\]
\[
a_x = 0, a_y = 0 \text{ and } f_k = \mu_k N \quad (3)
\]

From (1), (2), and (3)

\[
F = mg \frac{\sin \theta + \mu_k \cos \theta}{\cos \alpha - \mu_k \sin \alpha}
\]

\[
W = \vec{F} \cdot \vec{s} = (F \cos \alpha \hat{i} - F \sin \alpha \hat{j}) \cdot \hat{s} = Fs \cos \alpha
\]

\[
W = mgs \frac{\sin \theta + \mu_k \cos \theta}{1 - \mu_k \tan \alpha}
\]

(b) $\alpha = 0 \Rightarrow \tan \alpha = 0$ then

\[
W = mgs \sin \theta + \mu_k mgs \cos \theta
\]

The first term is the work done against the gravitation, the second term is the work done against friction.
Question 2 [Conservation of Energy]: A block of mass $m$ is released from an inclined plane as shown in Figure 2. The coefficient of kinetic friction between the inclined plane and the block is $\mu_k$. Assume that the spring is initially uncompressed and there is no friction on the horizontal surface.

(a) Find the maximum compression in the spring (10 points).

(b) If the block is bounced back from the spring, find the maximum height on the inclined plane that the block can reach (7 points).

(c) Check case (a) if $\mu_k = 0$ and comment on the result (3 points).

\[
E_i = U_i = mgd \sin \theta \quad (1)
\]

The speed of the block when it reaches the bottom of the inclined plane:

\[
\frac{1}{2}mv^2 = mgd \sin \theta - \mu_k mg \cos \theta \cdot d
\]

The maximum compression in the spring

\[
\frac{1}{2}kx^2 = \frac{1}{2}mv^2 \Rightarrow x = \sqrt{\frac{2mgd}{k} (\sin \theta - \mu_k \cos \theta)}
\]

(b) When the block reaches at maximum height on the inclined plane, $v = 0$

\[
\frac{1}{2}kx^2 = mg h_{max} - \mu_k mg \cos \theta \cdot \frac{h_{max}}{\sin \theta}
\]

\[
h_{max} = \frac{d(\sin \theta - \mu_k \cos \theta)}{1 + \mu_k \cot \theta}
\]

(c) If $\mu_k = 0$, then $h_{max} = d \sin \theta$, the block is returned back its initial height (Total energy is conserved).
**Question 3** [Momentum]: A man of mass \( m \) clings to a rope ladder suspended below a balloon of mass \( M \) as shown in Figure 3. The balloon is stationary with respect to the ground.

(a) If the man begins to climb the ladder at a speed \( v \) (with respect to the ladder), in what direction and with what speed (with respect to Earth) will the balloon move? (12 points).

(b) If the man then stops climbing, what is the speed of the balloon (with respect to Earth) (4 points)?

(c) Assuming that \( M >> m \), what is the speed of the balloon in case (a) (4 points)?

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**Solution 3:**

**Method 1: Center of mass**

(a) The net force on the balloon-man system is zero so the center of mass of this system is constant:

\[
\nu_{cm} = \frac{M\nu_b + m(\nu_b + v)}{m + M}
\]

\[
\nu_{cm} = 0 \Rightarrow \nu_b = -\frac{m}{m + M} v
\]

The balloon is moving downward.

(b) If \( v = 0 \Rightarrow \nu_b = 0 \), the balloon is stationary.

(c) If \( M >> m \Rightarrow \frac{m}{M} \rightarrow 0 \), \( \nu_b = -\frac{m/M}{M+1} v = 0 \): expected

**Method 2: Conservation of momentum**

(a) The momentum is conserved:

\[
P_i = 0, P_f = M\nu_b + m(\nu_b + v), P_i = P_f \Rightarrow \nu_b = -\frac{m}{m+M} v
\]
Question 4 [Collision]: The two balls, \( m_2 \) and \( m_3 \), on the right of Figure 4 are slightly separated and initially are at rest; the left ball, \( m_1 \), is incident on \( m_2 \) with speed \( v_0 \). Assuming head-on elastic collisions and no friction;

(a) Find speeds of \( m_1 \) and \( m_2 \) just after the first collision (8 points).
(b) Assuming that \( m_1 = m_2 = m \) and \( m_3 \leq m \), show that there are two collisions and find all final velocities (6 points).
(c) Assuming that \( m_1 = m_2 = m \) and \( m_3 > m \), show that there are three collisions and find all final velocities (6 points).

Solution 4:

(a) Due to elastic collision, momentum and energy are conserved:
\[
P_i = m_1 v_0, \quad P_f = m_1 v_1 + m_2 v_2 \\
E_i = \frac{1}{2} m_1 v_0^2, \quad E_f = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 \\
P_l = P_f \Rightarrow m_1 v_0 = m_1 v_1 + m_2 v_2 \quad (1) \\
E_l = E_f \Rightarrow \frac{1}{2} m_1 v_0^2 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 \quad (2)
\]
From (1) and (2)
\[
v_1 = \frac{m_1 - m_2}{m_1 + m_2} v_0 \quad (3) \\
v_2 = \frac{2m_1}{m_1 + m_2} v_0 \quad (4)
\]
(b) \( m_1 = m_2 = m, \ m_3 \leq m \)
After first collision between \( m_1 \) and \( m_2 \), \( v_1 = 0 \) and \( v_2 = v_0 \)
For the second collision, between \( m_2 \) and \( m_3 \), we do not need to solve the equations again, we can write directly from (3) and (4)
\[
v_2 = \frac{m - m_3}{m + m_3} v_0 \\
v_3 = \frac{2m}{m + m_3} v_0
\]
Since \( m_3 \leq m \), \( m_2 \) never returns back and collides with \( m_1 \), so we have only two collisions.

(c) Similarly, since \( m_3 \geq m \), \( m_2 \) returns back and collides with \( m_1 \). In this case, we have three collisions.
\[
v_1 = -\frac{m_3 - m}{m + m_3} v_0 \\
v_2 = 0 \\
v_3 = \frac{2m}{m + m_3} v_0
Question 5 [Angular Motion]: We could measure the speed of light, $c$, by using a rotating slotted wheel. As shown in the Figure 5, a beam of light passes through one of the slots at the outside edge of the wheel, travels to a mirror, and returns to the wheel just in time to pass through the next slot in the wheel. Assuming that the angular velocity of wheel is $w$, number of slots around wheel’s edge is $N$, and the distance between the wheel and the mirror is $L$, find the speed of light $c$ (20 points).

Solution 5:
In the time light takes to go from the wheel to the mirror and back again, the wheel turns through an angle

$$\Delta \theta = \frac{2\pi}{N}$$

The time is

$$\Delta t = \frac{2L}{c}$$

So the angular velocity of the wheel is

$$w = \frac{\Delta \theta}{\Delta t} = \frac{\pi c}{NL} \Rightarrow c = \frac{wNL}{\pi}$$