Date: November 4, 2004
Time: 10:40–12:30

NAME: ______________________________

STUDENT ID: ______________________________

MATH 220-01 MIDTERM I

IMPORTANT

1. This exam consists of 5 questions of equal weight.
2. Each question is on a separate sheet. Please read the questions carefully and write your answers under the corresponding questions. Be neat.
3. Show all your work. Correct answers without sufficient explanation might not get full credit.
4. Calculators are not allowed.

Please do not write anything below this line.

1	2	3	4	5	TOTAL

1. Solve the system and find a basis for the solution space of the corresponding homogeneous system:

$$\begin{array}{rcrcrcrcrcr}
x_1 & + & x_2 & - & x_3 & + & 2x_4 & - & x_5 & = & 7 \\
-2x_1 & + & 2x_2 & - & 6x_3 & & & - & 2x_5 & = & 2 \\
3x_1 & - & x_2 & + & 5x_3 & + & x_4 & - & x_5 & = & 6 \\
x_1 & - & x_2 & + & 3x_3 & - & x_4 & - & x_5 & = & 0
\end{array}$$

2. Find a basis for the space $V \subset P_5$ of polynomials p of degree up to 5 such that

$$p(2) - \frac{dp}{dt}(2) = \frac{d^2p}{dt^2}(2) = \frac{d^3p}{dt^3}(2) = 0.$$

What is the dimension of this space?

3. If exists, find A^{-1} for

$$A = \begin{bmatrix} 2 & 1 & 0 & 3 \\ 1 & 2 & 1 & 0 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix}.$$

4. Find the dimension and a basis for the subspace $\operatorname{Span} S \subset M_{2,2}$, where

$$S = \left\{ \begin{bmatrix} 3 & 2 \\ -1 & 2 \end{bmatrix}, \begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix}, \begin{bmatrix} 3 & 2 \\ -1 & 2 \end{bmatrix}, \begin{bmatrix} 5 & 6 \\ -3 & 2 \end{bmatrix}, \begin{bmatrix} 0 & 4 \\ -2 & -1 \end{bmatrix} \right\}.$$

5. There are 12000 undergraduate students at Bilkent. Let M be the (12000×12000)-matrix whose (i, j)-th entry is 1 if student i and student j have met each other and 0 if they have not met. What is the meaning of the (i, j)-th entry of M^2?