LINEAR ALGEBRA I

QUIZ 1

NAME: .....

(1) Explain why the set of all differentiable functions $y = f(x)$ satisfying $y' = y - 1$ do not form a vector space.

Any of the following observations does it:

(a) If f, g are in this space, then $(f + g)' = f + g - 2$, so $f + g$ is not in this space.
(b) If f satisfies $f'(x) = f(x) - 1$, then $r \cdot f$ does not (unless $r = 1$).
(c) This space does not conain the 0 function.

(2) Show that $\binom{1}{1}$ and $\binom{2}{1}$ are linearly independent.

Solve $a\binom{1}{1} + b\binom{2}{1} = 0$. We get the system of equations $a + 2b = 0$, $a + b = 0$, from which we easily find that $a = b = 0$.

(3) Write $\binom{3}{-1}$ as a linear combination of $\binom{1}{1}$ and $\binom{2}{1}$.

$\binom{3}{-1} = a\binom{1}{1} + b\binom{2}{1}$ gives the pair of linear equations $a + 2b = 3$, $a + b = -1$. This immediately shows $a = -5$, $b = 4$.