(1) Complete the following sentences: a linear map $L : U \rightarrow V$ is

- injective $(1 - 1)$ if and only if $\dim$ (1)
- surjective (onto) if and only if $\dim$ (15)
- an isomorphism if and only if $\dim$ (15)
(2) Let $U$ be a subspace of the inner product space, and let $u_1, u_2, u_3$ be a basis of $U$. Show that $v_1, v_2, v_3$ are pairwise orthogonal vectors in $U$, where

\[ v_1 = u_1, \]
\[ v_2 = u_2 - \frac{(u_2, v_1)}{(v_1, v_1)} v_1, \]
\[ v_3 = u_3 - \frac{(u_3, v_2)}{(v_2, v_2)} v_2 - \frac{(u_3, v_1)}{(v_1, v_1)} v_1. \]

(3) Show that three pairwise orthogonal vectors $v_1, v_2, v_3$ in an inner product space are always linearly independent.
(4) Use the Gram-Schmidt process to find an orthonormal basis of the inner product space $P_2$, where $(p, q) = \int_{-1}^{1} p(t)q(t)dt$. 
(5) Let $M_2(\mathbb{C})$ be the vector space of $2 \times 2$-matrices with complex entries, and define an inner product on $M_2(\mathbb{C})$ via $(A, B) = \text{Tr}(B^T A)$. Find an orthonormal basis for the subspace $W$ of $M_2(\mathbb{C})$ spanned by

\[ A = \begin{pmatrix} 1 & i \\ 0 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 1 + i & 0 \\ 1 & 0 \end{pmatrix}. \]
(6) Compute the determinant
\[
\begin{vmatrix}
ab & ac & 1 & bc \\
1 & a & 0 & a^2 \\
1 & b & 0 & b^2 \\
1 & c & 0 & c^2
\end{vmatrix}
\]

(7) a) Consider the map \( L : P_2 \longrightarrow P_2 \) defined by \( L(p) = p - tp' \). Compute \( \ker L \).

b) What is \( \dim \im L \)?
(8) a) For which values of $c$ does $(\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}) = x_1y_1 + x_1y_2 + x_2y_1 + cx_2y_2$ define an inner product on $\mathbb{R}^2$?

b) Find a vector orthogonal to $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ with respect to this inner product (for general $c$).