LINEAR ALGEBRA

MIDTERM 1, 20.10.2005

NAME: .....

(1) It is your responsibility to write clearly.
(2) Show all your work. Correct answers without sufficient explanation will not get full credit.
(3) Your answer should consist of complete sentences.

problem	1	2	3	4	5	6
points to earn	20	20	20	10	10	20
points earned						

(1) True or False? (No explanation required)

statement	true	false
Every nonzero matrix A has an inverse A^{-1}	☐	☐
The inverse of a product AB of square matrices A, B is equal to $A^{-1}B^{-1}$	☐	☐
Homogeneous linear systems of equations always have a solution	☐	☐
The rank of an $m \times n$-matrix is always $\leq n$	☐	☐
The set of polynomials of degree $= 2$ is a vector space	☐	☐
The product of an $m \times n$- and a $n \times k$-matrix is an $m \times k$-matrix	☐	☐
If $\{v_1, v_2, v_3\}$ is a basis of a vector space, then $\{v_1, v_2, v_3\}$ are linearly independent	☐	☐
If $\{v_1, v_2, v_3\}$ are linearly independent vectors in some vector space V, then they form a basis of V	☐	☐
The row rank of a matrix is equal to its column rank	☐	☐
For all 2×2-matrices A and B, we have $AB = BA$	☐	☐

(2) Compute the solution space of the homogeneous system $Ax = 0$ for

$$A = \begin{pmatrix} 2 & -1 & -2 \\ -4 & 2 & -4 \\ -8 & 4 & 8 \end{pmatrix}.$$

What is the rank of A?

(3) For which values of a does the inverse A^{-1} of

$$A = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 0 \\ 1 & 2 & a \end{pmatrix}$$

exist? Compute A^{-1} in these cases.

(4) Are the "vectors" $\left(\begin{smallmatrix}1&1\\1&1\end{smallmatrix}\right)$, $\left(\begin{smallmatrix}1&0\\0&2\end{smallmatrix}\right)$ and $\left(\begin{smallmatrix}0&1\\0&2\end{smallmatrix}\right)$ in the real vector space M_{22} of 2×2-matrices linearly independent?

(5) Let P be an $n \times n$-matrix with $P^2 = P$, let I denote the identity matrix of dimension n, and let $w \in \mathbb{R}^n$ be an arbitrary vector. Show that every vector $v = (P - I)w$ is a solution of the homogeneous system $Pv = 0$.

(6) a) Find a basis for the vector space of all polynomials p of degree ≤ 3 with $p(0) = p'(1) = 0$.

b) Write $p(x) = x^3 + 2x^2 - 7x$ as a linear combination of your basis.