

LINEAR ALGEBRA

HOMEWORK 8

- (1) Consider the linear map $L : \mathbb{R}_3 \rightarrow \mathbb{R}_3$ defined by

$$L(a, b, c) = (2a + 3b, -b + 4c, 3c).$$

Determine all eigenvalues and the associated eigenvectors.

- (2) Determine the characteristic polynomial, the eigenvalues, and the associated eigenvectors of

$$\begin{pmatrix} 2 & 1 & 2 \\ 2 & 2 & -2 \\ 3 & 1 & 1 \end{pmatrix}.$$

- (3) Let A be an upper triangular $n \times n$ -matrix (only 0s below the diagonal). Show that the eigenvalues of A are the entries on the diagonal.

- (4) Determine the eigenvalues and the bases of the associated eigenspaces for

$$A = \begin{pmatrix} 2 & 2 & 3 & 4 \\ 0 & 2 & 3 & 2 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

- (5) Let λ be an eigenvalue of a matrix A . Show that λ^2 is an eigenvalue of A^2 .