LINEAR ALGEBRA

HOMEWORK 7

(1) For a 2×2-matrix $A = \left(\begin{smallmatrix} a & b \\ c & d \end{smallmatrix}\right)$, define the transpose $A^T = \left(\begin{smallmatrix} a & c \\ b & d \end{smallmatrix}\right)$ and the trace $\mathrm{Tr}(A) = a + d$.

Last time you proved that $(A, B) = \mathrm{Tr}(B^T A)$ is an inner product in the vector space of 2×2-matrices with real entries. How do you have to modify the definition in order to get an inner product on the space of 2×2-matrices over the complex numbers? Check the axioms.

(2) Compute

$$\begin{vmatrix} a & b & c \\ c & a & b \\ b & c & a \end{vmatrix}.$$

(3) Assume that A is a square matrix with $A^2 = A$. Show that A is either singular, or that $\det A = 1$.

(4) Let A, B, C be 2×2-matrices. Show that

$$\det \begin{pmatrix} A & B \\ 0 & D \end{pmatrix} = \det A \det D.$$

(5) Assume that A is an $n \times n$-matrix with $A^T = -A$. Show that $\det A = 0$ whenever n is odd.