LINEAR ALGEBRA

HOMEWORK 5

(1) Find $a, b, c \in \mathbb{R}$ such that $v = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$ is orthogonal to $w = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$ and $x = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$
   (a) by solving the linear system of equations $v \cdot w = 0$, $v \cdot x = 0$;
   (b) by using the cross product.

(2) Let u, v, w be an orthonormal set of vectors in an inner product space. Compute $||u + v + w||^2$.

(3) Let V be the $\mathbb{R}$-vector space of polynomials with inner product $(p, q) = \int_0^1 p(t)q(t)dt$. For what values of a and b are $p(t) = 3t + 1$ and $q(t) = at + b$ orthogonal?

(4) Let V be the vector space of continuous real-valued functions on $[-\pi, \pi]$, and define an inner product on V by $(f, g) = \int_{-\pi}^{\pi} f(t)g(t)dt$. Show that the functions $\{1, \sin t, \cos t\}$ form an orthonormal set of vectors in V.

(5) Let V be an inner product space. Show that if $(u, v) = 0$ for all $v \in V$, then $u = 0$.