LINEAR ALGEBRA

HOMEWORK 3

(1) Find the 2×2-matrix corresponding to rotation by 180° in the plane $\mathbb{R}^2$.

(2) The map $L : \left(\begin{smallmatrix} x \\ y \end{smallmatrix}\right) \longmapsto \left(\begin{smallmatrix} x \\ 0 \end{smallmatrix}\right)$ projects each vector in $\mathbb{R}^2$ (vertically) onto the x-axis.
   (a) Show that L is a linear map $\mathbb{R}^2 \longrightarrow \mathbb{R}^2$.
   (b) Find the corresponding matrix P.
   (c) Show that $P^2 = P$ (here $P^2 = P \cdot P$).
   (d) Explain geometrically why $P^2 = P$.

(3) Give an example of a nonzero 3×3-matrix P with the property that $P^2 = P$.

(4) Let v_1, v_2, v_3 be vectors in some vector space V. Show that
   (a) $\text{span}(v_1, v_2, v_3) = \text{span}(v_1 + v_2, v_2, v_3)$.
   (b) If v_1, v_2, v_3 are linearly independent, then so are $v_1 + v_2, v_2, v_3$.