

LINEAR ALGEBRA

HOMEWORK 2

- (1) Write the polynomial $x + 1$ as a linear combination of the polynomials $2x^2 - x + 1$ and $-x^2 + x$.
- (2) Show that the vectors $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$, $\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$ are linearly independent.
- (3) For which values of c are the vectors $x + 3$ and $2x + c + 2$ in the vector space of polynomials of degree ≤ 2 linearly dependent?
- (4) Find a basis for all vectors of the form $(a+c, a-b, b+c, -a+b)$ for $a, b, c \in \mathbb{R}$. What is the dimension of this vector space? Does the vector $(3, 1, 2, -1)$ lie in this vector space? If yes, write it as a linear combination of your basis.
- (5) Consider the space $V = \text{span} \left\{ \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix}, \begin{pmatrix} 4 \\ 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} \right\}$. Find a basis for V , and determine its dimension.