

# LINEAR ALGEBRA I

## HOMEWORK 1

Which of the following spaces are vector spaces? Explain why or why not.

- (1) The set of all polynomials  $f$  of degree  $\leq 3$  with the property that  $f'(x) \leq 1$ .
- (2) The set of all polynomials  $f$  of degree  $\leq 3$  with the property that  $f'(x)$  is an integer.
- (3) The set of all triples  $(x, y, z)$  of real numbers with coordinatewise addition and scalar multiplication defined by  $r(x, y, z) = (rx, 0, rz)$ .
- (4) The set of all triples  $(x, y, z)$  of real numbers with coordinatewise addition and scalar multiplication defined by  $r(x, y, z) = (rx, y, rz)$ .
- (5) The set of real valued functions  $y = f(x)$  satisfying the differential equation  $y'' - y' + 2y = 0$ .
- (6) The set of real valued functions  $y = f(x)$  satisfying the differential equation  $y'' - y' + 2y - 1 = 0$ .

Note: In general, it is sufficient for you to check that vectors can be added, that there is a zero vector, and that there is a scalar multiplication. In spaces where the scalar multiplication is defined in some funny way (Ex. 3 and 4), however, you should carefully check all the axioms for scalar multiplication.