LINEAR ALGEBRA

PROBLEMS

1. Problems

(1) You should be able to solve essentially all the problems in Section 6.1 (Eigenvalues and Eigenvectors; Sect. 7.1 in the 8th ed.?) and Section 6.2 (Diagonalization; Sect. 7.2 in the 8th ed.?).

(2) Let $A = \left(\begin{smallmatrix} 0 & 1 \\ 0 & 0 \end{smallmatrix}\right)$. Show that A cannot be diagonalized. Hint: we did this in class; assume $D = P^{-1}AP$, show that $D = 0$, and derive a contradiction.

(3) Let $A = \left(\begin{smallmatrix} 1 & 1 \\ 1 & 0 \end{smallmatrix}\right)$. Show that A cannot be diagonalized. Hint: proceed as above; assume $D = P^{-1}AP$, show that $D = I$, and derive a contradiction.

(4) Show that the set of real symmetric $n \times n$-matrices form a subspace of $M_n(\mathbb{R})$.

(5) Consider the matrix

$$A = \begin{pmatrix} 2 & i & 3 \\ 1+i & 0 & 1-i \\ 2 & 1 & 2+i \end{pmatrix}.$$

Check that $\det A \neq 0$, and compute the inverse A^{-1}.

(6) Consider the matrix

$$A = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & i \\ 0 & -i & 2 \end{pmatrix}.$$

(a) Show that A is Hermitian, and conclude that the eigenvalues of A are real.
(b) Compute the eigenvalues and the associated eigenvectors of A.
(c) Find a matrix P such that $D = P^{-1}AP$ is diagonal.
(d) Compute P^{-1} and check $D = P^{-1}AP$ directly.

(7) Compute A^k for $A = P^{-1}DP$. What is A^{-1}?

2. Proofs

(1) Show that if λ is an eigenvalue of A, then λ^2 is an eigenvalue of A^2.

(2) Show that if λ is an eigenvalue of the nonsingular matrix A, then $\lambda \neq 0$, and λ^{-1} is an eigenvalue of A^{-1}.

(3) Show that A and $P^{-1}AP$, where P is nonsingular, have the same eigenvalues. (Thm. 6.2/7.2)

(4) Let A be a 3×3-matrix with distinct eigenvalues $\lambda_1, \lambda_2, \lambda_3$ and associated eigenvectors v_1, v_2, v_3. Show that v_1, v_2, v_3 are linearly independent, and therefore form a basis of $\mathbb{R}^3$. (Thm. 6.5/7.5)

(5) Show that symmetric matrices have real eigenvalues (Thm. 6.6/7.6). Give a direct proof for 2×2-matrices.

(6) Show that Hermitian matrices have real eigenvalues.