ALGEBRAIC NUMBER THEORY

MIDTERM 1

NAME: ....

problem	1	2	3	4	5
points to earn	20	20	20	20	20
points earned					

(1) Find the fundamental unit of $\mathbb{Q}(\sqrt{6})$.

(2) Show that $10 = 2 \cdot 5 = -\sqrt{-10} \cdot \sqrt{-10}$ is an example of nonunique factorization in $\mathbb{Z}[\sqrt{-10}]$, and show also how this factorization can be explained in terms of unique factorization into prime ideals.

(3) Let $p \equiv 3 \bmod 4$ be prime. Show that exactly one of the two equations $a^2 - 2pb^2 = 2$ or $a^2 - 2pb^2 = -2$ is solvable in integers.

(4) Write down all prime ideals of norm ≤ 11 in $K = \mathbb{Q}(\sqrt{103})$ (No proofs required, but your list should be correct and complete). Also find the prime ideal factorization of $(13 + \sqrt{103})$.

(5) Find the prime ideal factorizations of $(7 + \sqrt{103})$ and $(10 + \sqrt{103})$, and use this to write down an expression for a unit $\varepsilon > 1$ in $\mathcal{O}_K$ (you need not show that this unit is fundamental).