ALGEBRAIC NUMBER THEORY

HOMEWORK 5

(1) Show that $\mathrm{Im}\,(Mz) = \frac{\mathrm{Im}\,(z)}{|cz+d|^2}$ for $M = \left(\begin{smallmatrix} a & b \\ c & d \end{smallmatrix}\right)$.

(2) Show that $\mathrm{SL}_2(\mathbb{Z})$ acts on the upper half plane, i.e., that $(MN)z = M(Nz)$ for $z \in \mathcal{H}$.

(3) Show that $Q \sim Q|_M$ for $M \in \mathrm{SL}_2(\mathbb{Z})$ defines an equivalence relation on the set of binary quadratic forms of fixed discriminant Δ.

(4) Show that if a positive definite binary quadratic form Q corresponds to $z \in \mathcal{H}$, then for $M \in \mathrm{SL}_2(\mathbb{Z})$, the form $Q|_M$ corresponds to $M^{-1}z$.

(5) Show that the two binary quadratic forms $(1, 0, 3)$ and $(1, 1, 1)$ represent the same integers, but that they are not equivalent.

(6) Compute all reduced forms of discriminant $\Delta = -31$.