

ALGEBRAIC NUMBER THEORY

HOMEWORK 3

- (1) Show that if $m = 2p$ for some prime $p \equiv 5 \pmod{8}$, then the equation $x^2 - 2py^2 = -1$ has a solution in integers.
- (2) Show that if $m = 2p$ for some prime $p \equiv 5 \pmod{8}$, then the ring of integers \mathcal{O}_K in $K = \mathbb{Q}(\sqrt{m})$ does not have unique factorization. Hint: show that the prime ideals above 2 and p are not principal.
- (3) Compute the fundamental unit of $\mathbb{Q}(\sqrt{67})$ using elements of small norm and their prime ideal factorizations.
- (4) Determine whether the prime ideals above 3 in $\mathbb{Q}(\sqrt{229})$ are principal or not.