# ALGEBRAIC NUMBER THEORY

HOMEWORK 1

(1) Show that $6 = (1+\sqrt{-5}\,)(1-\sqrt{-5}\,) = 2\cdot 3$ is another example of nonunique factorization in $\mathbb{Z}[\sqrt{-5}\,]$. (You have to show that the factors above are irreducible and do not differ by units).

(2) Explain the different factorizations in Problem 1 using the ideals $\mathfrak{p} = (2, 1+\sqrt{-5}\,)$, $\mathfrak{q} = (3, 1+\sqrt{-5}\,)$, and $\mathfrak{q}' = (3, 1+\sqrt{-5}\,)$. Show that
   (a) $(2, 1-\sqrt{-5}\,) = \mathfrak{p}$;
   (b) $\mathfrak{p}^2 = (2)$;
   (c) $\mathfrak{q}\mathfrak{q}' = (3)$;
   (d) $\mathfrak{q}^2 = (2+\sqrt{-5}\,)$.

(3) Show that $6 = 2\cdot 3 = (2+\sqrt{-2}\,)(2-\sqrt{-2}\,)$ is not an example of nonunique factorization in $\mathbb{Z}[\sqrt{-2}\,]$.

(4) Compute the characteristic polynomial of $\alpha = a + b\sqrt[3]{2} + c\sqrt[3]{4}$ with respect to the $\mathbb{Q}$-basis $\{1, \sqrt[3]{2}, \sqrt[3]{4}\}$ of $\mathbb{Q}(\sqrt[3]{2})$.