

# ALGEBRAIC NUMBER THEORY

FINAL

NAME: .....

problem	1	2	3	4	5	6
points to earn	20	20	20	20	10	10
points earned						

- (1) Compute all reduced forms of discriminant  $\Delta = -4 \cdot 17$ . (The form  $(A, B, C)$  with  $A > 0$  is reduced if  $-A < B \leq A \leq C$ , with  $B > 0$  if  $A = C$ ).

(2) Compute the ideal class group of  $\mathbb{Q}(\sqrt{-17})$ .

Determine the ideal class to which the ideal  $(7, 2 + \sqrt{-17})$  belongs.

- (3) Compute the class group and the fundamental unit of  $K = \mathbb{Q}(\sqrt{10})$ .

- (4) Compute the class group and the fundamental unit of  $K = \mathbb{Q}(\sqrt{89})$ .

- (5) Assume that  $m \equiv 3 \pmod{4}$  is squarefree.
- (a) Show that  $[2, 1 + \sqrt{m}] = (2, 1 + \sqrt{m})$ .
  - (b) Find integers  $a$  and  $b$  such that  $[a, b + \sqrt{m}] \neq (a, b + \sqrt{m})$ , and explain why.

- (6) Explain how the ideal  $(2)$  splits in quadratic number fields  $\mathbb{Q}(\sqrt{m})$ .