ALGEBRAIC GEOMETRY

HOMEWORK 3

(1) Prove that $K[X,Y]/(XY) \simeq K[X] \oplus K[Y]$ as rings.

(2) An algebraic set $X \subseteq \mathbb{A}^n K$ is called reducible if there are algebraic sets $X_1, X_2 \neq X$ with $X = X_1 \cup X_2$, and irreducible otherwise.

Show that $\mathcal{V}(I)$ is irreducible in $\mathbb{A}^n K$ if and only if I is a prime ideal in $K[X_1, \ldots, X_n]$. (Hint: this is easy. Start writing down the definitions and think about what they imply.)

(3) Show that $\mathcal{V}(I)$ for $I = (XY) \subseteq K[X,Y]$ is reducible, and that $\mathcal{V}(J)$ for $J = (Y - X)$ is irreducible.

(4) Show that prime ideals are radical.