ALGEBRAIC GEOMETRY

HOMEWORK 1

(1) Find all rational points on the conic $X^2 + 2Y^2 = 3$.

(2) Use sing surf to sketch the 5-leaved rose: $(X^2 + Y^2)^3 - 5X^4Y + 10X^2Y^3 - Y^5 = 0$. Use the sweeping lines technique (start with $P = (0, 0)$ to find all rational points on this curve.

(3) Find all $\mathbb{Q}(T)$-rational points (points whose coordinates are fractions of polynomials in T) on the conic $X^2 - (T^4 - T^3)Y^2 = 1$.

(4) Show that $X^2 - (T^4 - T^3)Y^2 = 1$ does not have any nontrivial solutions in $\mathbb{Q}[T]$. Hint: Mason's theorem.