

ALGEBRAIC GEOMETRY

FINAL

NAME:

problem	1	2	3	4	5
points to earn	15	15	15	15	40
points earned					

- (1) Compute all singular points on the curve $y^4 = x^4 + 2x^3y$, and show that it can be parametrized.

- (3) (15) Let $V = \mathcal{V}(I)$ for $I = (y - x^2, z - x^3)$ be the “twisted cubic” in $\mathbb{A}^3\mathbb{R}$, and $W = \mathcal{I}(J)$ for $J = (v - u - u^2)$ a conic in $\mathbb{A}^2\mathbb{R}$. Show that $\phi : (x, y, z) \mapsto (u, v) = (xy, z + x^2y^2)$ is a polynomial map $V \rightarrow W$.

Let ϕ^* be the induced map $K[W] \rightarrow K[V]$. What is $\phi^*(u + J)$, $\phi^*(v)$ and $\phi^*(v - u - u^2)$?

- (4) Consider the curve $\mathcal{C} = \mathcal{V}(I)$ for $I = (Y^3 - X^4 + X^2)$, and let $f = \frac{x(x-1)}{y} \in K(\mathcal{C})$, where $x = X + I$ and $y = Y + I$.
- (a) Show that f is defined at $P = (1, 0)$.

(b) Show that f is not defined at $P = (0, 0)$.

- (5) (20) For $a \in \{1, 2, 3\}$ consider the curve $\mathcal{C} : y^2 = x^5 + x^a$ in the complex plane.
- (a) Determine $m_P(\mathcal{C})$ for $P = (0, 0)$ in each case.

(b) Determine $m_P(\mathcal{C})$ for the point at infinity.

(c) Compute the points of intersection for \mathcal{C} and the line $L : x = 0$.

(d) (continued from problem 5) Use problem 2c) to find lower bounds for the intersection multiplicities at the points of intersection. Explain what you are doing.

(e) Use Bezout's theorem to compute the intersection multiplicities exactly.

(f) Compute $I(P, \mathcal{C} \cap L)$ using the definition.