ALGEBRAIC GEOMETRY

HOMEWORK 6

(1) Let K be a field.
   (a) Show that $x^2 \in (x - y^2, xy)$ in $K[x, y]$.
   (b) Show that $(x - y^2, xy, y^2) = (x, y^2)$;
   (c) Is $(x - y^2, xy) = (x^2, xy)$? Justify your answer.

(2) An ideal I in a ring R is called radical if $\operatorname{rad} I = I$. Show that $\operatorname{rad} I$ is radical, i.e., $\operatorname{rad}(\operatorname{rad} I) = \operatorname{rad} I$.

(3) Show that if I and J are radical, then so is $I \cap J$.

(4) An affine variety V is called irreducible if $V = U \cup W$ for varieties U, W implies that $U = V$ or $W = V$. Show that V is irreducible if and only if $\mathcal{I}(V)$ is a prime ideal.