ALGEBRAIC GEOMETRY

HOMEWORK 4

(1) Show that the curve $y = \sin x$ is not algebraic. Do the same for $y = e^x$.

(2) Find the singular point on $y^2 = x^3 + x^2$ and the tangents at this point.

(3) Determine the multiplicity of the singular point $(0, 0)$ on $4x^2y^2 - (x^2+y^2)^3 = 0$.

(4) Let $\mathcal{C}$ be an irreducible curve of degree 5. Show that $\mathcal{C}$ does not have three collinear singular points.

(5) Determine the intersection multiplicity of *all* lines through the origin and the curve $\mathcal{C} : x^4 + y^4 + x^2 = 0$ in $P = (0, 0)$. How many tangents does $\mathcal{C}$ have in P?