ALGEBRAIC GEOMETRY

HOMEWORK 3

(1) Show that the set $\mathcal{C}(K) = \{[a^2 : ab : b^2] : (a, b) \in K \times K \setminus \{(0, 0)\}\}$ is a plane algebraic curve in $\mathbb{P}^2 K$. Also show that $\mathcal{C}$ is irreducible.

(2) Find all singular points on $\mathcal{C} : x^3 + y^3 + 1 + 3axy = 0$, where $a \in \mathbb{C}$.

(3) Find all singular points on the curve $x^4 + y^4 - x^2 y = 0$, and show that the curve can be parametrized.

(4) Compute the tangent of the real curve $x^3 + y^3 + 1 = 0$ at the (real) point at infinity.